HEAT TRANSFER DURING BOILING OF A FLUID ON CIRCUMFERENTIAL FINS

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Approximate solutions in generalized variables obtained for the temperature profile and thermal flux in an individual circumferential fin cooled by a boiling fluid are compared with experimental data for Freon 113.

A number of analytic [1-3] and experimental [3-5] papers have been devoted to a study of heat transfer during boiling of a fluid on the surface of individual fins of constant cross section. Little study has been given to fins of variable cross section. A limited amount of work was devoted to a study of heat transfer during boiling of a fluid in units with circumferential fins, although tubes with circumferential finning are often used in heat-exchange equipment (evaporators and condenser—evaporators). Heat transfer was investigated experimentally [6] for boiling of Freon 113 on tubes with circumferential finning of six sizes. The data obtained were compared with the results of numerical computer calculations.

We consider the one-dimensional equation of stationary thermal conductivity for a circumferential fin of variable cross section:

$$\mu_0^2 \frac{1}{R+\rho} \cdot \frac{d}{dR}\left\{\bar{\delta}(R)(R+\rho)\frac{d\Theta}{dR}\right\} = H(\Theta). \tag{1}$$

The boundary conditions are as follows:

$$\Theta = 1 \text{ for } R = 0,$$
$$\frac{d\Theta}{dR} = 0 \text{ for } R = 1, \tag{2}$$

where

$$\Theta = \frac{\vartheta}{\vartheta_0};\quad R = \frac{r-r_1}{r_2-r_1};\quad \rho = \frac{r_1}{r_2-r_1};\quad \bar{\delta} = \frac{\delta}{\delta_0};$$
$$\mu_0^2 = \frac{\lambda F_0}{\alpha_0 P_0 (r_2-r_1)^2} = \frac{\lambda\delta_0}{2\alpha_0 (r_2-r_1)^2};\quad H(\Theta) = \frac{\alpha(\Theta)}{\alpha_0}\Theta.$$

As shown in [7], with a small parameter present in the leading derivative (μ_0^2), it is possible to obtain an asymptotic approximation to the solution for Eq. (1) which is in the form of a series expansion in terms of the parameter μ_0*:

$$\Theta = f_0(\tau) + \mu_0 f_1(\tau) + \ldots, \tag{3}$$

where $\tau = R/\mu_0$.

* For example, the value of μ_0^2 does not exceed 0.3 for organic fluids and 0.04 for water during cooling of copper circumferential fins 3-4 mm thick and 20-30 mm high when ϑ_0 = 20°C.

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We expand the function $\bar{\delta}(R)$ in a Taylor series:

$$\bar{\delta}(R)=\bar{\delta}(0)+\bar{\delta}'(0)R+\ldots=1+\mu_0\gamma\tau+\ldots, \tag{4}$$

where $\gamma=\bar{\delta}'(0)$.

The right side of Eq. (1) can be represented as the series

$$H(\Theta)=H[f_0(\tau)+\mu_0 f_1(\tau)+\ldots]=H[f_0(\tau)]+\mu_0 f_1(\tau)H'_\Theta[f_0(\tau)]+\ldots. \tag{5}$$

Limiting ourselves to two terms of the expansion for simplicity of solution, substituting Eqs. (3), (4), and (5) into Eqs. (1) and (2), and equating terms with like powers of μ_0, we obtain equations and boundary conditions for the determination of the functions f_0 and f_1:

$$\left.\begin{aligned}&\frac{d^2f_0(\tau)}{d\tau^2}=H[f_0(\tau)]\\&f_0=1 \quad \text{for} \quad \tau=0\\&\frac{df_0}{d\tau}=0 \quad \text{for} \quad \tau=1/\mu_0\end{aligned}\right\}, \tag{6}$$

$$\left.\begin{aligned}&\frac{d^2f_1(\tau)}{d\tau^2}-H'_\Theta[f_0(\tau)]f_1(\tau)=-\frac{d}{d\tau}\left[\gamma\tau\frac{df_0(\tau)}{d\tau}+\frac{f_0(\tau)}{\rho}\right]\\&f_1=0 \quad \text{for} \quad \tau=0\\&\frac{df_1}{d\tau}=0 \quad \text{for} \quad \tau=1/\mu_0\end{aligned}\right\}. \tag{7}$$

Although the function $H(\Theta)$ itself is nonlinear in this case, a solution of Eq. (6) which does not contain the argument τ on the right side is determined in quadratures. The equations for succeeding functions (f_1 etc.) are linear, and therefore their solutions can often be obtained in final form.

We consider the application of the method discussed above for the case of bubbling boiling over the entire surface of a fin.*

For the majority of fluids, $\alpha(\vartheta)\sim\vartheta^2$ $[H(\Theta)=\Theta^3]$. Integrating Eq. (6) over the limits of variation of the functional values at the base and top of the fin, we obtain

$$\frac{1}{2}\left(\frac{df_0}{d\tau}\right)^2=\int_{f_0^h}^{f_0}H(f_0)\,df_0. \tag{8}$$

Since the value of $(f_0^h)^4$ can be neglected $((f_0^h)^4\ll f_0^4$ [8]), we have

$$\frac{df_0}{d\tau}=-\frac{f_0^2}{\sqrt{2}}, \tag{9}$$

$$f_0=\frac{1}{1+\left(\frac{\tau}{\sqrt{2}}\right)}. \tag{10}$$

Using Eq. (10) for the solution of Eq. (7), we obtain

$$\frac{1}{2}\cdot\frac{d^2f_1}{du^2}-\frac{3}{u^2}f_1=\frac{\gamma\sqrt{2}}{u^3}-\frac{\gamma}{\sqrt{2}u^2}+\frac{1}{\sqrt{2}\rho u^2}, \tag{11}$$

where $u=1+(\tau/\sqrt{2})$.

The general solution of Eq. (11) is

$$f_1=C_1u^3+C_2u^{-2}-\frac{\gamma u^{-1}}{\sqrt{2}}+\frac{1}{3\sqrt{2}}\left(\gamma-\frac{1}{\rho}\right). \tag{12}$$

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* The zone of free convection has little effect on the total thermal flux and is not taken into consideration here.

Using the boundary conditions (7), we find approximate values for the constants C_1 and C_2:

$$C_1 = 0; \quad C_2 = \frac{1}{3}\left(\gamma\sqrt{2} + \frac{1}{\rho\sqrt{2}}\right).$$

Substituting Eqs. (10) and (12) into Eq. (3), we obtain

$$\Theta = u^{-1} + \mu_0\left[\frac{u^{-2}}{3}\left(\gamma\sqrt{2} + \frac{1}{\rho\sqrt{2}}\right) - \frac{\gamma u^{-1}}{\sqrt{2}} + \frac{1}{3\sqrt{2}}\left(\gamma - \frac{1}{\rho}\right)\right]. \tag{13}$$

For a circumferential fin of constant thickness, Eq. (13) transforms to

$$\Theta = u^{-1} + \frac{\mu_0}{3\sqrt{2}\rho}(u^{-2} - 1). \tag{14}$$

The thermal flux through the base of the fin is

$$q_0 = -\lambda\frac{d\vartheta}{dr}\bigg|_{r=r_1} = -\frac{\lambda\vartheta_0}{\sqrt{2}\mu_0(r_2 - r_1)} \cdot \frac{d\Theta}{du}\bigg|_{u=1} = \vartheta_0\sqrt{\frac{\alpha_0\lambda}{\delta_0}}\left[1 + \mu_0\frac{\sqrt{2}}{3}\left(\frac{1}{\rho} + \frac{\gamma}{2}\right)\right]. \tag{15}$$

For the case of bubbling boiling of a fluid on the surface of a circumferential fin of constant thickness ($\gamma = 0$), a second approximation of the corresponding expansion (3) was calculated. The function $f_2(\tau)$ satisfies the equation

$$\frac{d^2 f_2(\tau)}{d\tau^2} - H'_\Theta[f_0(\tau)]f_2(\tau) = \frac{1}{2}H''_{\Theta^2}[f_0(\tau)][f_1(\tau)]^2 + \frac{\tau}{\rho^2}\cdot\frac{df_0(\tau)}{d\tau} - \frac{1}{\rho}\cdot\frac{df_1(\tau)}{d\tau}. \tag{16}$$

The boundary conditions are

$$f_2 = 0 \text{ for } \tau = 0, \quad \frac{df_2}{d\tau} = 0 \text{ for } \tau = \frac{1}{\mu_0}. \tag{17}$$

The solution for the function f_2 has the form

$$f_2 \cong \frac{1}{18\rho^2}(u^{-3} + 5u - 6) \tag{18}$$

and the thermal flux density through the base of the fin is

$$q_0 = \vartheta_0\sqrt{\frac{\alpha_0\lambda}{\delta_0}}\left[1 + \frac{\sqrt{2}}{3}\cdot\frac{\mu_0}{\rho} - \frac{1}{9}\left(\frac{\mu_0}{\rho}\right)^2\right]. \tag{19}$$

For a circumferential fin of constant thickness ($\gamma = 0$), computed relations were obtained for the coexistence of three boiling zones on the fin: bubbling boiling ($\alpha \sim \vartheta^2$ for $\vartheta < \vartheta_{cr_1}$), a zone of critical thermal flux in which the thermal flux density is assumed constant and equal to q_{cr} ($\alpha \sim 1/\vartheta$ for $\vartheta_{cr1} \le \vartheta \le \vartheta_1$), and a transitional zone ($\alpha \sim \vartheta^{-4}$ for $\vartheta > \vartheta_1$). For a nonisothermal surface, such approximation by power functions is close to the boiling curves for Freon 113 and water (up to $\vartheta_0 = 100°C$) [9]. Final expressions are presented below for the thermal flux density through the base of the fin which were obtained by using the first approximation (intermediate calculations omitted):

$$q_0 = \vartheta_{cr1}\sqrt{\alpha_{cr1}\frac{\lambda}{\delta}}K_i\left(1 + \frac{\mu}{\rho}\Pi_i\right), \tag{20}$$

where i is the number of the zone at the base of the fin.* In contrast to Eq. (15), it is more appropriate to take the critical parameters ϑ_{cr1} and α_{cr1} as characteristic parameters in the case of two or three boiling zones:

$$\frac{\mu}{\rho} = \frac{1}{r_1}\sqrt{\frac{\lambda\delta}{2\alpha_{cr1}}} \quad \text{and} \quad \Theta_0 = \frac{\vartheta_0}{\vartheta_{cr1}}.$$

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* In the case of one-sided cooling, δ is understood to mean twice the thickness of the fin.

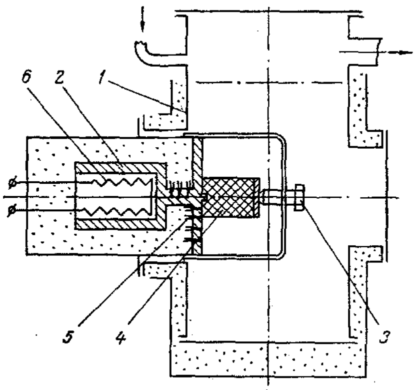

Fig. 1

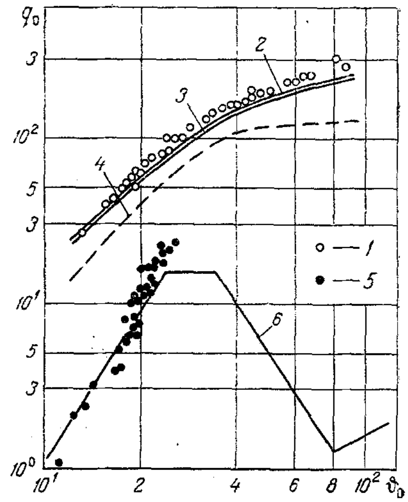

Fig. 2

Fig. 1. Diagram of experimental device.

Fig. 2. Dependence of thermal flux density on temperature head during boiling of Freon 113: 1, 5) experimental data for circumferential fin and isothermal surface, respectively; 2, 3) from Eq. (20) and computer calculation for a circumferential fin, respectively; 4) from Eq. (20) for a straight fin; 6) approximation to data of [9], q_0, W/cm²; ϑ_0, °C.

For $\Theta_0 \leq 1$ (i = 1),

$$K_1 = \Theta_0^2; \quad \Pi_1 = \frac{\sqrt{2}}{3} \cdot \frac{1}{\Theta_0}.$$

For $1 \leq \Theta_0 \leq \Theta_1$ ($\Theta_1 = \vartheta_1/\vartheta_{cr1}$; i = 2),

$$K_2 = \sqrt{4\Theta_0 - 3}; \quad \Pi_2 = \frac{K_2^3 + 1}{3\sqrt{2}\,K_2^2}.$$

For $\Theta_0 > \Theta_1$ (i = 3)

$$K_3 = \sqrt{4\Theta_1 - 3 + 2\Theta_1^3\left(\frac{1}{\Theta_1^2} - \frac{1}{\Theta_0^2}\right)},$$

$$\Pi_3 = \frac{\Theta_0 - \Theta_1}{\sqrt{2}\,K_3} + \frac{\Pi_2(\Theta_1) + \dfrac{\Theta_0 - \Theta_1}{\sqrt{2}\,K_3}}{1 + \dfrac{2(\Theta_0 - \Theta_1)}{4\Theta_1 - 3}}.$$

Equation (20) with $\rho = \infty$, which determines the thermal flux transferred by a straight fin, is similar to the expression obtained in [8] for so-called "asymptotic" fins, for which the thermal flux through the base is practically independent of the length of the fin.

Experimental study of heat transfer during boiling of Freon 113 on a circumferential fin at atmospheric pressure was carried out on the device diagrammed in Fig. 1. The face of the copper block 2, a disk 2.2 mm thick with an outer diameter of 140 mm, was introduced into the stainless steel tank 1. The Teflon cylinder 4 (30 mm in diameter) was pressed against the disk by means of the screw 3. The fluid boiled on the circumferential surface of the disk not covered by the cylinder. This simulated a single circumferential fin cooled from one side on a supporting tube 30 mm in diameter. Temperatures on the cylindrical measuring section, at various points on the disk, and in the volume of the fluid were determined with the Chromel–Alumel thermocouples 5 (0.2 mm in diameter). The copper block was heated by the electrical heater 6. An additional heater not shown in the figure was introduced into the volume of the fluid in order to maintain it at saturation temperature. The arrows indicate vapor exit and condensate entrance.

The thermal flux was determined from the readings of the thermocouples on the cylindrical measuring section and in the disk beneath the Teflon cylinder 4. The observed discrepancy of 10% was associated with

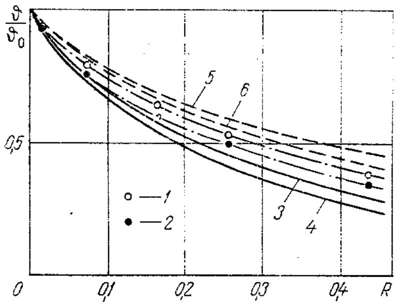

Fig. 3

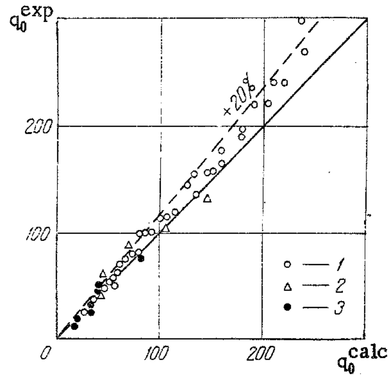

Fig. 4

Fig. 3. Dimensionless temperature as a function of dimensionless radius. Experimental data: 1) $q_0 = 53$, $\vartheta_0 = 18$; 2) $q_0 = 78.5$, $\vartheta_0 = 23.1$; computed data: 3, 4) from Eq. (14) for $\vartheta_0 = 18$ and 23.1, respectively; 5, 6) for a straight fin, from [3], $\vartheta_0 = 18$ and 23.1. q_0, W/cm^2; ϑ_0, °C.

Fig. 4. Comparison of experimental and computed thermal flux densities for Freon 113 boiling on circumferential fins: 1) data from this work: $r_1 = 15$, $\delta = 2.2$, $h = 55$; 2, 3) data from [6]: $r_1 = 12.7$, $\delta = 0.58$, $h = 9.5$ and $r_1 = 15.9$, $\delta = 5.1$, $h = 25.4$, respectively. q, W/cm^2; r_1, δ, and h are in mm.

flow along the Teflon cylinder (1-1.5%) and with the error in temperature determination. The maximum relative error in the determination of q_0 was ±20-25% corresponding to an error of ±10-15% in the determination of ϑ_0.

Control experiments were also performed on the boiling of Freon 113 on an isothermal surface; for this, an assembly in which the face of a copper block 35 mm in diameter served as the heated surface was introduced into the tank 1.

Figure 2 presents experimental data for an isothermal surface and a circumferential fin with one-sided cooling in the temperature-head range encompassing the modes of bubbling boiling and transitional boiling (up to $\vartheta_0 = 80$°C). Also shown is curve 2 calculated from Eq. (20). For this, the local thermal flux density for boiling on a nonisothermal surface, which was obtained in [9], was approximated by the following relations: $q = 1.1 \cdot 10^{-3}\vartheta^3$ for $\vartheta \leq 23.9$ (region of bubbling boiling); $q = 15$ for $23.9 \leq \vartheta \leq 35$ (critical region); $q = 6.44 \cdot 10^5 \cdot \vartheta^{-3}$ for $35 \leq \vartheta \leq 70.6$ (region of transitional boiling); these are indicated by the line 6. The results of computer calculations (by the Runge—Kutta method) performed by us with the same boundary conditions for a circumferential fin with an external diameter of 140 mm (external diameter of the disk) are shown by curve 3 ($\mu^2 = 0.0441$, $\mu/\rho = 0.77$).

As is evident, the experimental data are 10-15% higher than the results of the calculation based on Eq. (20) and 15-20% higher than the results of the numerical calculation.

The thermal flux density q_0 calculated from Eq. (19) is approximately 5% below the corresponding values obtained from Eq. (20) and is in practical agreement with the results of the numerical calculation.

When $\rho = \infty$, Eq. (20) degenerates into a relation for "asymptotic" straight fins (curve 4). In the region of large temperature heads, the deviation of curve 4 from the experimental data for a circumferential fin reaches 80-90%.

One should note the relative stability of the thermal flux density at the base of a fin with respect to a temperature shift in the function $\alpha(\vartheta)$ approximating the boiling curve. The very nature of local boiling curves (presence of a bubbling boiling zone where $dq/d\nu > 0$ and of transitional boiling where $dq/d\nu < 0$) leads to compensation of the reduction in heat transfer in some sections by its increase in others in the case of a temperature shift (for large temperature heads at the base of a fin). In addition, $q_0 \sim \alpha^{0.5}$ in the case of small μ_0 so that an insignificant change in $\alpha(\vartheta)$ leads to a still smaller change in q_0.

Figure 3 shows experimental temperature profiles and profiles calculated from Eq. (14) for bubbling boiling of Freon 113 on the surface of a circumferential fin. Temperature profiles for a straight fin calculated in [3] are also shown (curves 5 and 6). In the region of small dimensionless radii R, the experimental and computed values of ϑ/ϑ_0 are close, which is evidence of the good agreement between calculated and experimental values for the thermal flux density at the base of a fin. The divergence increases in proportion to the increase in R and reaches 30-35%.

It is of interest to see how well calculation by the proposed method agrees with the experimental data of other authors. Because of the absence of data on heat transfer from individual circumferential fins, the results in [6] for finned tubes were used to calculate heat transfer from an individual fin located on the tube. To do this, the thermal flux from the smooth portion of the tube, which was also studied in [6], was subtracted from the total thermal flux released by a fin and by the portion of the supporting wall between fins.

Figure 4 shows a comparison of experimental data from the present work for an "asymptotic" fin and of data calculated from [6] for individual, nearly "asymptotic" fins [parameters: $r_1 = 15.9$, $h = 25.4$, $\delta = 5.08$ mm (tube A) and $r_1 = 12.7$, $h = 9.5$, $\delta = 0.584$ mm (tube H)] with calculated data based on Eq. (20). The approximate relations for Freon 113 boiling on a nonisothermal surface given above were used in the calculation. The large gap between fins on tubes A and H (13.9 and 4.5 mm, respectively) eliminated its effect on heat transfer both from the fin and from the portion of the supporting tube between fins. Experimental data were no more than 20-25% higher than calculated data over the entire range of thermal flux densities.

NOTATION

q, thermal flux density; ϑ, r, r_1, δ, temperature head, fin radius, fin base radius, and fin thickness; Θ, R, ρ, $\bar{\delta}$, dimensionless temperature head, fin radius, fin base radius, and fin thickness; r_2, h, F, P, external radius, height, cross-sectional area, and perimeter of fin; λ, α, coefficients of thermal conductivity and heat transfer;

$$H = \left[\frac{\alpha(\vartheta)}{\alpha_0}\right]\Theta;\quad \mu_0 = \frac{1}{r_2 - r_1}\sqrt{\frac{\lambda\delta_0}{2\alpha_0}};$$

$\tau = R/\mu_0$, dimensionless parameters; f_0, f_1, f_2 and K, Π, functions of τ and Θ_0, Θ_1, respectively. Indices: 0, fin base; h, fin top; cr, critical; calc, calculated; exp, experiment.

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